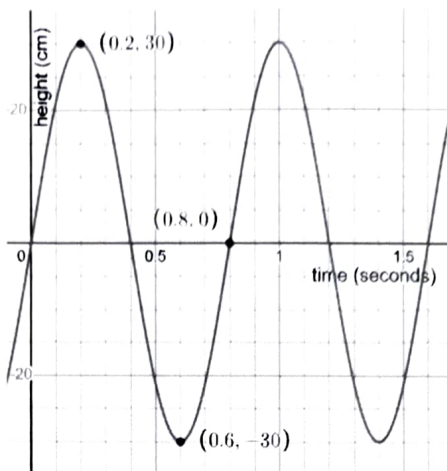


Unit Circle Problem-solving Practice Questions

Question 1 (1 + 2 + 1 + 2 + 1 + 1 = 8 marks)

Marking Key

Lisa attaches a tag onto one of the spokes at the rim of Bart's bike as he rides at a constant speed. She records the height of the tag with respect to its starting position with a video.



The vertical axis represents the height (cm) of the tag from its starting position with the video and the horizontal axis represents the time (seconds) since Bart started riding.

- a) How long does it take for the wheel to rotate once?

0.8s (1)

- b) Find the equation of the graph above

A = 30

T = \frac{2\pi}{n}

\frac{2\pi}{5} = \frac{2\pi}{n}

n = \frac{5}{2}\pi

T = 0.8

y = 30 \sin \frac{5}{2} \pi t

- c) What is the radius of the wheel?

r = 30 cm (1)

- d) Calculate the circumference of the wheel.

C = 2\pi r = 60\pi cm (1)

2(\frac{22}{7})(30) = \frac{1320}{7} cm

- e) How would you expect the amplitude of the graph to change if Lisa repeated the experiment with her dad's bike- which has a larger wheel?

Higher / Larger amplitude (1)

- f) How would you expect the graph to change if Bart started riding the bike faster?

The distance between two consecutive crests / troughs / points will be shorter (1) (1 + 2 + 1 + 2 + 1 + 1 = 8 marks)
OR
any reasonable answers.



Question 2 (2 + 2 + 2 + 1 = 7 marks)

A weather station models the daily temperature variation using the function $y = 12 \sin(3\theta)$, where y represents the deviation in temperature (in degrees Celsius) from the average temperature throughout the day, and θ is the time in hours from midnight.

- a) What is the amplitude of this function, and what does it represent in this context?

$$A = 12 \text{ , deviation of } \pm 12^\circ\text{C}$$

- b) How many complete cycles of temperature variation occur in 24 hours? Explain your reasoning.

$$\because n = 3 \quad \therefore 3 \text{ complete cycles.}$$

- c) Calculate the period of the function and explain what it represents in this context.

$$n = 3 \quad T = \frac{2\pi}{n} = \frac{2}{3}\pi \quad \text{the cycle repeats itself every } \frac{2\pi}{3} \text{ hours.}$$

- d) Determine the frequency of this function in terms of the number of cycles per hour.

$$f = \frac{1}{T} = \frac{1}{\frac{2\pi}{3}} = \frac{3}{2\pi} \text{ cycles per hour.}$$

Question 3 (1 + 1 + 1 + 1 = 4 marks)

A company tracks the daily power consumption using the function $y = 8 \sin(2\theta)$, where y represents the deviation from the average power consumption in kilowatts, and θ is the time in hours from midnight.

- a) What does the amplitude of 8 represent in this context?

$$8 \text{ kW}$$

- b) How many complete cycles of power consumption occur in a day

$$2 \text{ complete cycles}$$

- c) Calculate the period of the function.

$$n = 2 \quad T = \frac{2\pi}{n} = \frac{2\pi}{2} = \pi$$

- d) If the company wants to minimize power consumption during peak hours (between 6 AM and 9 AM), suggest a strategy based on the function provided. What changes might indicate a successful implementation of this strategy?

$$\text{Reduce/decrease the amplitude.}$$

Question 4 (2 + 1 + 1 + 1 = 5 marks)

A research team models the daily hours of sunlight in a remote area using the function $y = 10 \cos(4\theta)$, where y represents the deviation in hours of sunlight from the average throughout the day, and θ is the time in hours from midnight.

- a) What is the amplitude of this function, and what does it represent in this context?

$A = 10$ ①, deviation in daily hours of ① sunlight.

- b) How many complete cycles of sunlight variation occur in 24 hours?

4 complete cycles ①

- c) Calculate the period of the function.

$$n = 4 \quad T = \frac{2\pi}{n} = \frac{2\pi}{4} = \frac{\pi}{2} \text{ ①}$$

- d) Determine the frequency of this function in terms of the number of cycles per hour.

$$f = \frac{1}{T} = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi} \text{ cycles per hour. ①}$$

Question 5 (2 + 4 + 4 = 10 marks)

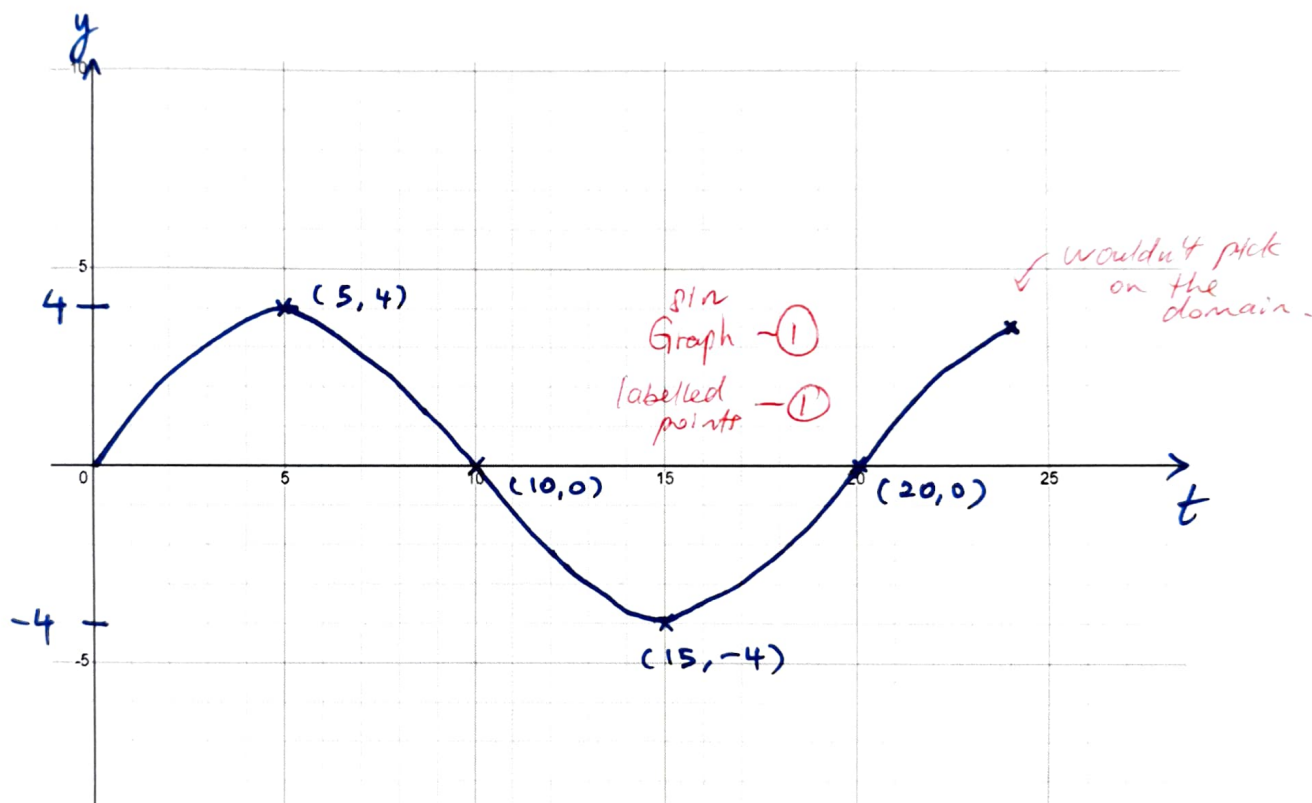
The height of tides in a particular coastal area is modeled by the function $y = 4\sin \frac{\pi}{10}t$, where y is the height of the tide in metres and t is the time in hours from midnight, $0 \leq t \leq 24$.

- a) Identify the amplitude and explain its significance in terms of tide height.

$A = 4$ (1), max tide is 4m. (1) x.

- b) Sketch one complete cycle of this function over the course of 24 hours. Label the amplitude, period, and key points on your sketch.

$A = 4$ (1) $n = \frac{\pi}{10}$ $T = \frac{2\pi}{\frac{\pi}{10}} = 2\pi \times \frac{10}{\pi} = 20$ (1)



- c) If a ship needs at least 2 meters of water to dock safely, during which hours of the day can the ship safely dock?

when $y = 2$

$$2 = 4\sin \frac{\pi}{10}t$$

$$\frac{1}{2} = \sin \frac{\pi}{10}t$$

$$\sin \frac{\pi}{10}t = \frac{1}{2}$$

also accept between 1:40a.m. — 8:20a.m. or after 9:40p.m.

between $1\frac{2}{3}$ and $8\frac{1}{3}$ after $21\frac{2}{3}$

$$\frac{\pi}{10}t = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\frac{\pi}{10}t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

$$t = \frac{\pi}{\frac{\pi}{10}}, \frac{5\pi}{\frac{\pi}{10}}, \frac{13\pi}{\frac{\pi}{10}}, \frac{17\pi}{\frac{\pi}{10}}$$

$$= \frac{5}{3}, \frac{25}{3}, \frac{65}{3}, \frac{85}{3}$$

$$1\frac{2}{3}, 8\frac{1}{3}, 21\frac{2}{3}, 28\frac{1}{3}$$

rejected! past 24 hrs.