East Doncaster Secondary College

EDSC East Doncaster

Year 9 ALPHA Mathematics

Semester 1 Exam, 2022

Name: PROPOSED MARKING SCHEME

Teacher (Circle): RAB XUE

Date: _____

Reading Time: 10 Minutes

Writing Time: 90 Minutes

Section	Type	Questions	Total Marks for Section
A	Multiple Choice	10	10
B	Short Answer	13	45
C	Extended Response	1	5
			60

Information:

- Students **are not permitted** to bring mobile phones and/or any other unauthorised electronic devices into the examination room
- Students **are permitted** to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers and one bound reference book.
- Students **are not permitted** to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- A **scientific calculator** is allowed in this exam
- EAL students are allowed to bring into the examination room a hard copy dictionary; electronic dictionaries are **NOT** allowed.
- Express answers to 2 decimal places where necessary unless instructed otherwise.
- **Please fill in the boxes at the bottom of Page 2 to answer the Multiple-Choice Section**

Section A – Multiple Choice

Question 1

The gradient of the line which passes through the points $(1,8)$ and $(3,-6)$ is

A. 7

B. -7

C. $\frac{7}{2}$

D. $\frac{-7}{2}$

Question 2

Which of the following has a solution of $x = -5$?

A. $3x - 7 = 8$

B. $\frac{2x+3}{13} = 1$

C. $\frac{12}{x} = -2.4$

D. $\frac{3}{2-x} = 1$

Question 3

The pair of linear equations, $y = x + 3$ and $y = -2x + 6$, intersect at the point

A. $(-3,0)$

B. $(3,6)$

C. $(1,4)$

D. $(-1,8)$

Question 4

Which of the following is irrational

A. $2\sqrt{5}$

B. $\sqrt[3]{64}$

C. $\sqrt[5]{32}$

D. $2\sqrt{100}$

Question 5

Which of the following is not equivalent to $\sqrt{48}$

A. $2\sqrt{12}$

B. $\sqrt{8} \times \sqrt{6}$

C. $4\sqrt{3}$

D. $2\sqrt{6} \times 2\sqrt{2}$

Question 6

What is the factorised form of $3(a-2) - b(a-2)$

A. $(a-2)(3-b)$

B. $(a-2)(b-3)$

C. $(a-2)(3+b)$

D. $(a+2)(3+b)$

Question 7

The solutions to $x^2 - 18 = 0$ are:

A. $x = -3\sqrt{2}, 3\sqrt{2}$

B. $x = -4\sqrt{2}, 4\sqrt{2}$

C. $x = -2\sqrt{3}, 2\sqrt{3}$

D. $x = -18, 18$

Question 8

The factorised form of $x^2 + 6x - 7$ is:

A. $(x+3)^2 - 16$

B. $(x-3)^2 - 16$

C. $(x+3)^2 + 16$

D. $(x-3)^2 + 16$

Question 9

The exact area of the slanted *surface* of a cone with radius 2 cm and slant height 6 cm is:

A. $12\pi\ cm^2$

B. $18\pi\ cm^2$

C. $3\pi\ cm^2$

D. $6\pi\ cm^2$

Question 10

A sphere has a volume equal to its surface area (in value). The radius of the sphere is:

A. 1 unit

B. 2 units

C. π units

D. 3 units

Question	1	2	3	4	5	6	7	8	9	10
Response	B	C	C	A	D	A	A	A	A	D

Section B – Short Answer

Question 1 *(2 + 2 = 4 Marks)*

Solve each of the following for x

a) $3x + 5 = 5x + 11$

$-2x = 6$ (1M)

$x = 3$ // (1A)

b) $\frac{x-3}{3} = \frac{5-x}{4}$

$\therefore 4(x-3) = 3(5-x)$

$\therefore 4x - 12 = 15 - 3x$ (1M - EXPANSION)

$\therefore 7x = 27$

$\therefore x = 27/7$ // (1A)

Question 2 *(2 + 3 = 5 Marks)*

Solve each of the following inequalities for x

a) $3 - \frac{5x}{7} \leq -2$

$\therefore -\frac{5x}{7} \leq -5$ (1M - TRANSPOSE)

$\therefore -5x \leq 35$

$\therefore x \geq 7$ // (1A - MUST FLIP SIGN)

b) $\frac{5x-6}{3} - \frac{1-2x}{4} \geq 3x + 4$

$\therefore 4(5x-6) - 3(1-2x) \geq 12(3x+4)$

$\therefore 20x - 24 - 3 + 6x \geq 36x + 48$ (1M)

$\therefore 26x - 27 \geq 36x + 48$

$\therefore -10x \geq 75$ (1M)

$\therefore x \leq -75/10$

$\therefore x \leq -15/2$ // (1A)

Question 3 *(2 Marks)*

Find the exact distance between the pair of points (–2, 4) and (–7, –4). $x_1\ y_1 \quad x_2\ y_2$

$d = \sqrt{(-4-4)^2 + (-7+2)^2}$ (1M)

$\therefore \sqrt{(-8)^2 + (-5)^2}$

$\therefore \sqrt{89}$ UNITS (1A MUST INCLUDE 'UNITS')

Question 4 *(2 + 2 + 2 = 6 Marks)*

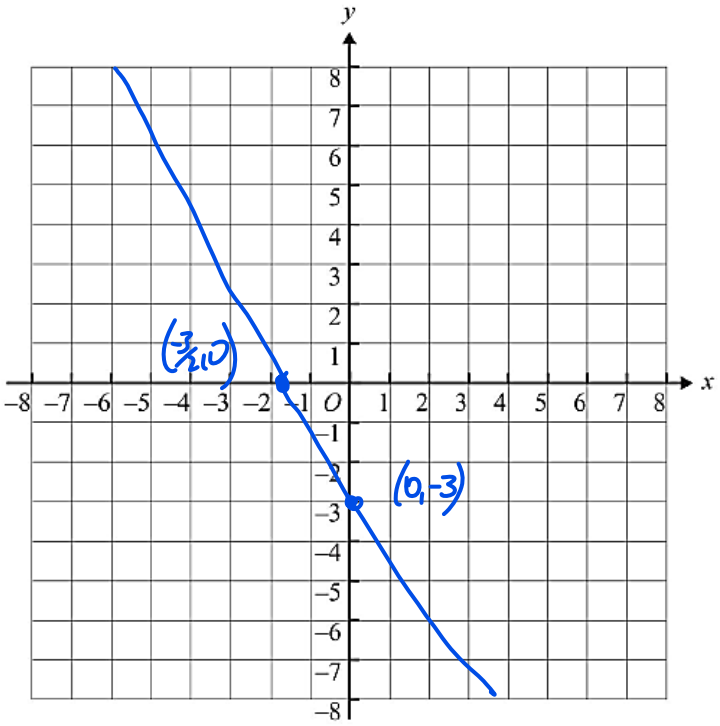
a) Sketch the graph of $y = -2x - 3$, labelling all axes intercepts

X-INT

$0 = -2x - 3$

$\therefore 3 = -2x$

$\therefore x = -\frac{3}{2}$



1A - SHAPE

1A - LABELLED AXES INTERCEPTS

b) State the equation of the line perpendicular to $y = -2x - 3$, which passes through $(-\frac{3}{2}, 6)$

$m_2 = \frac{1}{2}$ 1M - $\perp$ LM IDENTIFIED

$\therefore y - y_1 = \frac{1}{2}(x - x_1)$

$\therefore y - 6 = \frac{1}{2}(x + \frac{3}{2})$

$\therefore y = \frac{1}{2}x + \frac{3}{4} + 6$

$\therefore y = \frac{1}{2}x + \frac{27}{4}$ 1A

NOTE: NO CONSEQUENTIAL MARKS

c) Show that the equation, $3y + 6x = -7$ would have no solutions with the equation in part **a)**.

$y = -2x - 3$

$\therefore 3y + 6x = -7 \equiv y = -2x - \frac{7}{3}$ 1M - REARRANGE EQUATION

$m_1 = m_2$

$\therefore$ NO SOLUTIONS AS LINES ARE PARALLEL 1A

Question 5 *(2 Marks)*

Calculate the midpoint of the line segment joining the points (–3, 1) and (5, 3)

$M = \left(\frac{-3+5}{2}, \frac{1+3}{2}\right)$ 1M

$\therefore M = \left(\frac{2}{2}, \frac{4}{2}\right)$

$M = (1, 2)$ 1A

Question 6 *(3 Marks)*

Solve the following pair of simultaneous equations

$$6y + 3x = -9$$
$$5x - 3y = 11 \quad \times 2$$

$6y+3x=-9$ (1)

$+ \quad -6y+10x=22$ (3)

$13x = 13$

$x = 1$ 1M

Sub $x=1$ into (1)

$\therefore 6y+3=-9$

$y=-2$ 1M

P.O.I $= (1,-2)$ 1A

Question 7 *(1 + 2 = 3 Marks)*

Factorise each of the following

a) $x^2 + 2x - 63$

$(x+9)(x-7)$ 1A

b) $2x^2 + 7x + 3$

$\therefore 2x^2+6x+x+3$

$\therefore 2x(x+3)+1(x+3)$ 1M

$\therefore (2x+1)(x+3)$ 1A

Question 8 *(2 Marks)*

Simplify the following expression

$$\frac{x^2 - 8x - 20}{x^2 + 10x + 21} \div \frac{x^2 - 100}{x^2 - 3x - 18}$$

$\therefore \frac{(x-10)(x+2)}{(x+7)(x+3)} \times \frac{(x-6)(x+3)}{(x-10)(x+10)}$ 1M – FACTORISE ALL QUADRATICS

$= \frac{(x+2)(x-6)}{(x+7)(x+10)}$ 1A

Question 9 *(3 Marks)*

Using the method of 'Completing the Square', solve for the values of 'x'

$$2x^2 + 12x - 1 = 0$$

$\therefore 2x^2 + 12x = 1$

$\therefore x^2 + 6x = \frac{1}{2}$

$\therefore x^2 + 6x + 9 = \frac{1}{2} + 9$ 1M

$\therefore (x+3)^2 = \frac{19}{2}$

$\therefore x + 3 = \pm\sqrt{\frac{19}{2}}$

$\therefore x = -3 \pm \frac{\sqrt{19}}{2}$ 1M

$\therefore x = \frac{-3 \pm \sqrt{38}}{2}$ // 1A – MUST BE RATIONALISED

Question 10 *(2 Marks)*

Solve the following using the quadratic formula

$$4x^2 - 12x + 1 = 0$$

$x = \frac{12 \pm \sqrt{144 - 16}}{8}$

$x = \frac{12 \pm \sqrt{128}}{8}$ 1M

$\therefore x = \frac{12 \pm 8\sqrt{2}}{8}$

$x = \frac{3 \pm 2\sqrt{3}}{2}$ // 1A

Question 11 *(2 Marks)*

Show that $4x^2 + kx + 1 = 0$, has only one solution, when $k = \pm 4$

$\Delta = b^2 - 4ac$, $\Delta = 0$, FOR 1 SOLN (1M)

$\therefore k^2 - 16 = 0$

$\therefore k^2 = 16$

$\therefore k = \pm 4$ // (1A)

Question 12 *(3 + 3 = 6 Marks)*

Simplify each of the following using positive indices

a) $\dfrac{15x^2y^4}{2xy} \div \dfrac{(5xy^3)^2}{4(xy)^{-1}}$

$\therefore \dfrac{15xy^3}{2} \div \dfrac{25x^2y^6}{4x^4y^{-1}}$ (1M - EXPAND)

$\therefore \dfrac{15xy^3}{2} \times \dfrac{4x^{-3}y^{-7}}{25}$ (1M - APPLY INDEX LAW)

$\therefore \dfrac{6}{5x^2y^4}$ (1A - ARRANGED WITH POSITIVE INDICES)

b) $(8x^6y)^{\frac{1}{3}} \times 9x^{\frac{1}{5}}y^{\frac{2}{3}}$

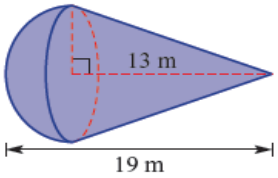
$\therefore 8^{\frac{1}{3}}x^2y^{\frac{1}{3}} \times 9x^{\frac{1}{5}}y^{\frac{2}{3}}$ (1M - EXPAND)

$\therefore 2x^2y^{\frac{1}{3}} \times 9^{\frac{1}{5}}y^{\frac{2}{3}}$ (1M)

$\therefore 18x^{\frac{11}{5}}y$ (1A)

Question 13 *(1 +1 + 3 = 4 Marks)*

This composite 3D solid comprises of a hemisphere and a cone, as shown.



Leave all answers in exact form unless told otherwise.

a) Determine the radius of the hemisphere

6m 1A

b) Calculate the exact slant height of the cone.

$s = \sqrt{36+169}$

$s = \sqrt{205}$ m 1A

c) Calculate the total surface area of this object. Express your answer in exact value.

TSA $\pi rs + \frac{1}{2}(4\pi r^2)$ 1M

$= (\pi \times 6 \times \sqrt{205}) + (2 \times \pi \times 6^2)$ 1M

$= 6\sqrt{205}\pi + 72\pi$

$= (6\sqrt{205}+72)\pi \ m^2$ 1A } MUST BE IN EXACT VALUE

NOTE: ACCEPT BOTH ANSWERS

Section C - Extended Response

Question 1 *(2 + 3 = 5 Marks)*

A cylinder has a radius of $\left(\frac{1}{\sqrt{2}-1}\right) cm$ and a height of $(\sqrt{2}+1)cm$

a) Show that the volume of this cylinder can be expressed as $\left(\frac{\pi(\sqrt{2}+1)}{3-2\sqrt{2}}\right)$ cm^3

$V = \pi r^2 \times h$

$= \pi \left(\frac{1}{\sqrt{2}-1}\right)^2 \times (\sqrt{2}+1)$

$= \frac{\pi(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}-1)}$ (1M) - EXPANDED CORRECTLY

$= \frac{\pi(\sqrt{2}+1)}{2-2\sqrt{2}+1}$

$V = \frac{\pi(\sqrt{2}+1)}{3-2\sqrt{2}}\ cm^3$ (1A)

b) Hence or otherwise, show that the volume of the cylinder is exactly $\pi(7+5\sqrt{2})cm^3$

$V = \frac{\pi(\sqrt{2}+1)}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$ (1M)

$\therefore V = \frac{\pi(\sqrt{2}+1)(3+2\sqrt{2})}{9-8}$ (1M)

$\therefore V = \pi(\sqrt{2}+1)(3+2\sqrt{2})$

$\therefore V = \pi(3\sqrt{2}+4+3+2\sqrt{2})$

$\therefore V = \pi(7+5\sqrt{2})\ cm^3$ (1A)

END OF EXAMINATION