2023 Semester 1

Year 10 Mathematical Methods

Coordinate Geometry Modelling Task

Name: Solutions Mark: /30 Teacher's code: _____

Time Allowed: *40 minutes writing time. No reading time.*

Instructions:

- *This is a* ***Technology allowed*** *task so CAS and a scientific calculator are allowed*
- ***One*** *double sided A4 page of summary notes may be used*
- *For questions worth* ***one mark, only the answer is required***
- *For questions where* ***more than one mark*** *is available,* ***appropriate working must be shown***
- ***Exact answers*** *are required unless otherwise stated*

Useful Formula

Midpoint formula	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Distance formula	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Equation of a straight line	$y = mx + c$ $y - y_1 = m(x - x_1)$
Gradient of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$

Question 1

A straight line passes through the points $A(3,7)$ and $B(2,-2)$.

a) Find the gradient of $\overline{AB}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2-7}{2-3} = 9$ (1) MARK (1) Mark

(2 marks)

b) Find the equation of $\overline{AB}$ and express it in gradient intercept form ($y = mx + c$).

$y - y_1 = m(x - x_1)$

Sub $(2,-2)$

$y + 2 = 9(x-2)$ (1) mark

$y = 9x - 20$ (1) mark

(2 marks)

Question 2

Find the gradient and y-intercept of the following linear equation.

$$y + \frac{1}{3} = -2(x-4)$$

$y = -2x + 8 - \frac{1}{3}$

$= -2x + \frac{23}{3}$

gradient : -2 (1) mark

y-intercept : $\frac{23}{3}$ (1) mark

3

(2 marks)

Question 3

Determine the equation of a line that is perpendicular to $y = \frac{1}{8}x + 2$ and passes through the point $(1,-5)$.

$m_\perp = \frac{-1}{\frac{1}{8}} = -8$ } (1) mark

$y + 5 = -8(x-1)$

$y = -8x + 3$ (1) mark

(2 marks)

Question 4

The sum of Declan and Annika's ages is 24 years, and the difference between their ages is 6 years. Find their ages given that Declan is older than Annika.

$d + a = 24$
$d - a = 16$ } (1) mark

$d = 15$
$a = 9$ } (1) mark

(2 marks)

Question 5

GWSC gardening club has planted eight two-metre-tall eucalyptus trees in their garden. The trees have grown at an average rate of $\frac{3}{4}$ metres per year. At the same time, they decide to plant three one-meter-tall Blueberry ash trees which grow twice as fast as the eucalyptus trees.

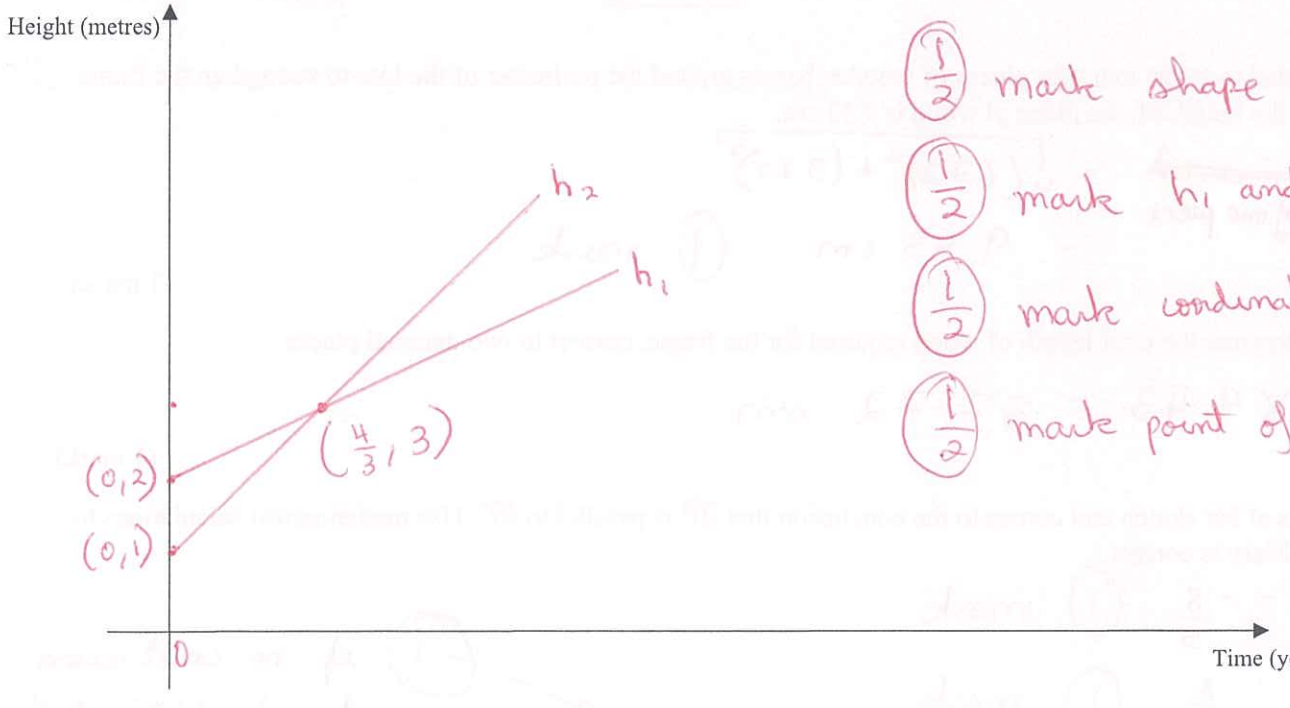
a) The equation $h_1 = 2 + \frac{3}{4}t$ gives the height of the eucalyptus trees in terms of the number of years, t.
Show that the linear model h_2 that gives the height of the blueberry ash trees in terms of the number of years, t, is given by: $h_2 = 1 + \frac{3}{2}t$.

$m_{h_2} = 2 \times m_{h_1}$
$= 2 \times \frac{3}{4}$
$= \frac{3}{2}$ (1) mark

1 meter is initial height
$\therefore c = 1$ (1) mark

(2 marks)

b) On the axis below sketch and label both models, h_1 and h_2. Label all axes intercepts and the point of intersection with their coordinates.



$\frac{1}{2}$ mark shape
$\frac{1}{2}$ mark h_1 and h_2
$\frac{1}{2}$ mark coordinates of y-intercept
$\frac{1}{2}$ mark point of intersection

(2 marks)

c) After how many months will the blueberry ash trees be the same height as the eucalyptus trees?

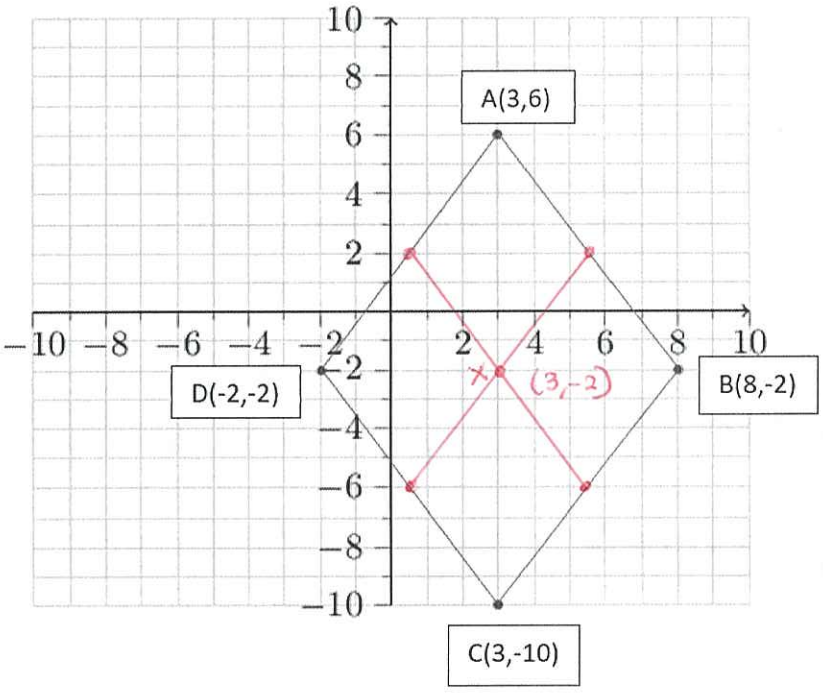
$12 \times \frac{4}{3} = 16$ months (1) mark

(1 mark)

Question 6

Mary would like to design a kite with equal side lengths for the school fare. She decides to sketch the kite on graphing paper but does not know how to proceed. An image of Mary's sketch is shown below.

Figure 1.1



a) Mary decided to insert four thin pieces of wooden beams around the perimeter of the kite to strengthen the frame. Show that the length of one piece of wood is 9.43 cm.

~~Total wood~~ Length of one piece $= \sqrt{(6+2)^2 + (3+2)^2}$

$= 9.43$ cm (1) mark

(1 mark)

b) Hence, determine the total length of wood required for the frame, correct to two decimal places.

$4 \times 9.43 = 37.72$ cm

(1 mark)

c) Mary looks at her sketch and comes to the conclusion that $\overline{AB}$ is parallel to $\overline{DC}$. Use mathematical calculations to show that Mary is correct.

$m(\overline{AB}) = -\frac{8}{5}$ (1) mark

$m(\overline{DC}) = -\frac{8}{5}$ (1) mark

(-1) if no conclusion due to same gradient

Since gradients are the same Mary is correct.

(2 marks)

d) In constructing the kite, Mary decides to insert one wooden beam from the midpoint of $\overline{AD}$ to the midpoint of $\overline{BC}$, and another wooden beam from the midpoint of $\overline{AB}$ to the midpoint of $\overline{CD}$.

i. The midpoint of $\overline{AD}$ is $\left(\frac{1}{2}, 2\right)$ and the midpoint of $\overline{BC}$ is $\left(\frac{11}{2}, -6\right)$. Using your CAS or otherwise, determine the midpoints of $\overline{AB}$ and $\overline{CD}$. (No working required)

$\overline{AB}$: $\left(\frac{11}{2}, 2\right)$ (1) mark

$\overline{CD}$: $\left(\frac{1}{2}, -6\right)$ (1) mark

(2 marks)

ii. Determine the equation of the wooden beam connecting the midpoint of $\overline{AD}$ and the midpoint $\overline{BC}$. Give your answer in the form $y = mx + c$.

$m = \frac{-6-2}{\frac{11}{2} - \frac{1}{2}} = -\frac{8}{5}$

Sub $\left(\frac{1}{2}, 2\right)$

$y - 2 = -\frac{8}{5}\left(x - \frac{1}{2}\right)$ } (1) mark

$y = -\frac{8}{5}x + \frac{14}{5}$ (1) mark

(2 marks)

e)

i. Sketch the two wooden beams on Figure 1.1 provided on the previous page.

No marks if ~~to~~ beams drawn outside kite.

(1 mark)

ii. Label the coordinates of the point of intersection, ***X***, on Figure 1.1 on the previous page.

(1 mark)

f) Determine the total length of the two new beams correct to two decimal places.

All beams are the same length } (1) mark

$\therefore 2(9.43)$

$= 18.86$ cm (1) mark

(2 marks)

g) Mary decides to connect a piece of string from X to the midpoint of $\overline{XC}$. Find the length of the piece of string.

4 cm (1) mark

(1 mark)

Having finished the design of the kite, Mary must determine the cost of constructing it. An equation for the total cost, C, is given below where w represents the amount of wood (in cm) and s represents the amount of string in cm.

$$C = 0.20s + 1.22w$$

h) How much will the construction of this kite cost Mary? Give your answer to the nearest dollar.

Total wood : $37.72 + 18.86 = 56.58$ cm

Total string : 4 cm

$C = 0.20(4) + 1.22(56.58)$ (1) mark

$= 69.83$

$= \$70$ (1) mark

(2 marks)

End of the CAT ☺